TRANSITIONAL PROCESS IN A COUNTERFLOW THERMODIFFUSION APPARATUS WITH CLOSED STREAM MOTION

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The concentration of the mixture is determined as a function of time and the parameters of the apparatus.

In [1] relations were obtained describing the transitional process in a counterflow thermodiffusion apparatus with closed stream motion for a narrow range of variation of the concentration of the mixture being separated, when one can take c(1 − c) = const.

It is interesting to examine the separation process in such an apparatus in larger concentration regions, including such important cases as the cleaning out of a small admixture and enrichment with an initial low content of the component, when c(1 − c) ≈ a + bc.

A schematic diagram of the motion of streams of the mixture to be separated is presented in Fig. 1. On the basis of the model formulated earlier [1], an equation is obtained describing the variation of the concentration,

$$m^* \frac{\partial c}{\partial t} = -\frac{\partial \tau}{\partial z}, \tag{1}$$

with the boundary conditions

$$\tau^* \bigg|_{y=y_e} = \sigma_e \frac{\partial c}{\partial x}\bigg|_{y=y_e}, \quad \tau^* \bigg|_{y=0} = \sigma_i \frac{\partial c}{\partial x}\bigg|_{y=0}. \tag{2}$$

The transport in the direction of the z axis per unit width of the column is

$$\tau^* = H^* \left[c(1-c) - \frac{\partial c}{\partial y} \right], \tag{3}$$

where

$$H^* = \frac{H}{B} = \frac{\alpha\rho^2 g\beta\delta^3 (\Delta T)^2}{6!\eta\overline{T}}; \quad y = \frac{Hz}{K}; \quad K = \frac{g^2\rho^3\beta^2\delta^7 (\Delta T)^2 B}{9!\,\eta^2 D}.$$

In the closed scheme represented in Fig. 1, the conditions (2) can be supplemented by the equalities of the concentrations at the entrance to and exit from the apparatus at the points of turning of the stream of the mixture to be separated:

$$(c_e - c_i)_{x=0} = 0, \quad (c_e - c_i)_{x=B} = 0. \tag{4}$$

Let us consider the region of concentrations in which the linear approximation

$$c(1-c) = a + bc \tag{5}$$

is valid, where a and b are constants. We introduce the dimensionless variables

$$u = c - c_0, \quad \omega = \frac{M}{mL}, \quad \theta = \frac{H^2 t}{mK}, \quad \xi = \frac{x}{B}, \quad \varkappa = \frac{\sigma}{HB}. \tag{6}$$

With allowance for (3)-(6), the initial relations (1) and (2) take the form

$$\frac{\partial u}{\partial \theta} = \frac{\partial^2 u}{\partial y^2} - b\frac{\partial u}{\partial y}, \tag{7}$$

$$\left[\frac{\partial u}{\partial y} + \varkappa \frac{\partial u}{\partial \xi} - bu \right]_{y=y_e} = a + bc_0, \tag{8}$$

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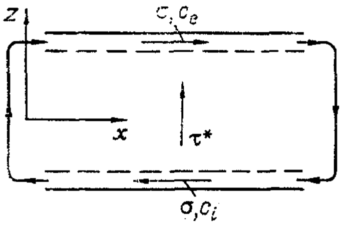

Fig. 1. Diagram of stream motion in the column.

$$\left[\frac{\partial u}{\partial y}+\varkappa\frac{\partial u}{\partial \xi}-bu\right]_{y=0}=a+bc_0. \tag{9}$$

As the initial condition we take

$$u|_{\theta=0}=0. \tag{10}$$

We apply a Laplace transformation with respect to the time variable to (7), and for the transforms we find

$$\bar{u}=A\exp\left(\frac{b}{2}y\right)\operatorname{ch}\alpha y+B\exp\left(\frac{b}{2}y\right)\operatorname{sh}\alpha y, \tag{11}$$

where it must be kept in mind that $A = A(\xi)$ and $B = B(\xi)$. Determining $A(\xi)$ and $B(\xi)$ from (8) and (9) and substituting them into (11), we obtain

$$\begin{aligned}\bar{u}=&\,C_1\exp\left(\frac{b}{2\varkappa}\xi+\frac{b}{2}y\right)\operatorname{ch}\left(\frac{\alpha}{\varkappa}\xi-\alpha y\right)+C_2\exp\left(\frac{b}{2\varkappa}\xi+\frac{b}{2}y\right)\times\\&\times\operatorname{sh}\left(\frac{\alpha}{\varkappa}\xi-\alpha y\right)+\frac{a+bc_0}{p^2\operatorname{sh}\alpha y_e}\left(\alpha\left(\exp\left(\frac{b}{2}(y-y_e)\right)\operatorname{ch}\alpha y-\right.\right.\\&\left.-\exp\left(\frac{b}{2}y\right)\operatorname{ch}(\alpha y_e-\alpha y)\right)+\frac{b}{2}\left(\exp\left(\frac{b}{2}y\right)\operatorname{sh}(\alpha y_e-\alpha y)+\exp\left(\frac{b}{2}y-\frac{b}{2}y_e\right)\operatorname{sh}\alpha y\right)\Bigg).\end{aligned} \tag{12}$$

To determine the constants C_1 and C_2 we use the conditions (4), giving

$$C_1=\frac{\alpha(a+bc_0)}{p^2\operatorname{sh}\alpha y_e\operatorname{sh}\dfrac{\alpha}{\varkappa}}\left(\left(\exp\left(\frac{b}{2}y_e\right)\operatorname{sh}\left(\frac{\alpha}{\varkappa}-\alpha y_e\right)-\operatorname{sh}\frac{\alpha}{\varkappa}\right)\exp\left(\frac{b}{2\varkappa}\right)+\exp\left(\frac{b}{2}y_e\right)\operatorname{sh}\alpha y_e\right), \tag{13}$$

$$\begin{aligned}C_2=&\frac{\alpha(a+bc_0)}{p^2\operatorname{sh}\alpha y_e\operatorname{sh}\dfrac{\alpha}{\varkappa}}\left(\exp\left(\frac{b}{2}y_e\right)\operatorname{ch}\alpha y_e-1-\right.\\&\left.-\exp\left(\frac{b}{2\varkappa}\right)\left(\exp\left(\frac{b}{2}y_e\right)\operatorname{ch}\left(\frac{\alpha}{\varkappa}-\alpha y_e\right)-\operatorname{ch}\frac{\alpha}{\varkappa}\right)\right).\end{aligned} \tag{14}$$

Substituting (13) and (14) into (12), we obtain the solution for the transforms, which at the exits from the upper and lower channels at $\xi = 1$ and $\xi = 0$ takes the form

$$\bar{u}_{ek}=\frac{(a+bc_0)b}{2p^2}+\frac{(a+bc_0)\alpha}{p^2\operatorname{sh}\dfrac{\alpha}{\varkappa}}\left(\operatorname{ch}\frac{\alpha}{\varkappa}-\exp\left(\frac{b}{2\varkappa}\right)\right), \tag{15}$$

$$\bar{u}_{ik}=\frac{(a+bc_0)b}{2p^2}-\frac{(a+bc_0)\alpha}{p^2\operatorname{sh}\dfrac{\alpha}{\varkappa}}\left(\operatorname{ch}\frac{\alpha}{\varkappa}-\exp\left(-\frac{b}{2\varkappa}\right)\right), \tag{16}$$

where the subscript k corresponds to the values of $\bar{u}$ at the exit from the apparatus.

Using the expansion theorem, we obtain

$$\frac{a+bc_{ek}}{a+bc_0}=\frac{b}{\varkappa\left(1-\exp\left(-\frac{b}{\varkappa}\right)\right)}+\frac{2b}{\varkappa}\exp\left(\frac{b}{2\varkappa}\right)\times$$

$$\times\sum_{n=1}^{\infty}\frac{\pi^2n^2}{\left(\frac{b^2}{4\varkappa^2}+\pi^2n^2\right)^2}\left[(-1)^n-\exp\left(-\frac{b}{2\varkappa}\right)\right]\exp\left(-\frac{\frac{b^2}{\varkappa}+4\pi^2n^2}{\frac{4}{\varkappa}}\theta\right), \tag{17}$$

$$\frac{a+bc_{ik}}{a+bc_0}=\frac{b}{\varkappa\left(\exp\left(\frac{b}{\varkappa}\right)-1\right)}+\frac{2b}{\varkappa}\sum_{n=1}^{\infty}\frac{\pi^2n^2}{\left(\frac{b^2}{4\varkappa^2}+\pi^2n^2\right)^2}\times$$

$$\times\left[1+(-1)^{n+1}\exp\left(-\frac{b}{2\varkappa}\right)\right]\exp\left(-\frac{\frac{b^2}{\varkappa^2}+4\pi^2n^2}{\frac{4}{\varkappa^2}}\theta\right). \tag{18}$$

From Eqs. (17) and (18) it follows that in a steady state in a counterflow thermodiffusion apparatus with closed motion of the streams, the concentration is determined by the relation

$$\frac{a+bc_{ek}}{a+bc_{ik}}=\exp\left(\frac{b}{\varkappa}\right). \tag{19}$$

In the case of cleaning, when the concentration of the main component is close to unity, $a = 1$, $b = -1$, and the expression for the degree of separation takes the form

$$q=\frac{1-c_{ik}}{1-c_{ek}}=\exp\left(\frac{1}{\varkappa}\right). \tag{20}$$

A comparison of (17)-(20) with the corresponding relations describing the variation of the concentration in a thermodiffusion column closed at both ends [2] shows that if

$$y_e=\frac{1}{\varkappa}, \tag{21}$$

where y_e is the height parameter of the column closed at both ends while $\varkappa$ is the parameter of pumping rate in the closed counterflow apparatus, then the kinetic curves and the degrees of separation in the steady state in these apparatus coincide. A decrease in the parameter $\varkappa$ leads to an increase in the degree of separation. According to the definition (6), the dependence of $\varkappa$ on the pumping rate and the size of the apparatus has the form

$$\varkappa=\frac{\sigma}{H_0B\delta^3}, \tag{22}$$

where

$$H_0=\frac{\alpha g\beta\rho^2(\Delta T)^2}{6!\,\eta\overline{T}}.$$

It is seen from Eqs. (19), (20), and (22) that, by varying the pumping rate σ, the length B of the apparatus, and the working gap δ, one can significantly influence the degree of separation in a counterflow apparatus with closed motion of the streams, with the degree of separation increasing with increase in the distance between the hot and cold surfaces. This fact is of practical interest, but it must be considered that an increase in the gap is limited by the conditions of the laminar convection regime in the working volume of the apparatus.

NOTATION

α, thermodiffusion constant; β, volumetric expansion coefficient; δ, gap, i.e., distance between the heated and cooled surfaces; ρ, density; σ, mass rate of liquid pumping through the channels; η, coefficient of dynamic viscosity: B, length of the apparatus; D, diffusion coefficient; L, height of the apparatus; T_2, T_1, temperatures of the heated and cooled surfaces; $\Delta T = T_2 - T_1$; $\overline{T} = 1/2(T_2+T_1)/2$; z, vertical coordinate; x, longitudinal coordinate; c, mass concentration; t, time. Indices: e, upper channel; i, lower channel; 0, initial value.

LITERATURE CITED

1. A. V. Suvorov and G. D. Rabinovich, "Transitional process in a thermodiffusion apparatus with transverse streams," Inzh.-Fiz. Zh., 39, No. 1, 86-96 (1980).
2. G. D. Rabinovich, Fractionation of Isotopes and Other Mixtures by Thermodiffusion [in Russian], Atomizdat, Moscow (1981).

SOLUTION OF ONE PROBLEM OF HEAT CONDUCTION IN A REGION WITH A MOVING BOUNDARY BY THE METHOD OF EXPANSION IN ORTHOGONAL WATSON OPERATORS

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The problem of heat conduction with branching of the heat flux at moving boundaries is solved by the method of expansion in orthogonal Watson operators.

The problem of the temperature distribution along two linear heat conductors with thermally insulated lateral surfaces is considered. We assume that a linear combination of the unknown functions and their derivatives is assigned at the moving ends of these heat conductors, and the ends of a heat conductor move by a linear law. This problem comes down to the solution of the system [1]

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{a^2}\frac{\partial u}{\partial t}, \quad \frac{\partial^2 v}{\partial y^2} = \frac{1}{b^2}\frac{\partial v}{\partial t} \tag{1}$$

in the region $\ell_1 + pt \le x \le \ell_2 + pt$, $\ell_2 - \ell_1 > 0$; $\ell_3 + qt \le y \le \ell_4 + qt$, $\ell_4 - \ell_3 > 0$, $-\infty < t < +\infty$, with the following initial (at $t = -\infty$) and boundary conditions at the boundaries moving by the linear law:

$$u|_{t=-\infty} = 0, \quad v|_{t=-\infty} = 0, \tag{2}$$

$$\left(\alpha_{11}\frac{\partial u}{\partial x} + \alpha_{12}u\right)\Big|_{x=l_1+pt} + \left(\alpha_{13}\frac{\partial v}{\partial y} + \alpha_{14}v\right)\Big|_{y=l_3+qt} = h_1(t), \tag{3}$$

$$\left(\alpha_{21}\frac{\partial u}{\partial x} + \alpha_{22}u\right)\Big|_{x=l_1+pt} + \left(\alpha_{23}\frac{\partial v}{\partial y} + \alpha_{24}v\right)\Big|_{y=l_4+qt} = h_2(t), \tag{4}$$

$$\left(\alpha_{31}\frac{\partial u}{\partial x} + \alpha_{32}u\right)\Big|_{x=l_2+pt} + \left(\alpha_{33}\frac{\partial v}{\partial y} + \alpha_{34}v\right)\Big|_{y=l_3+qt} = h_3(t), \tag{5}$$

$$\left(\alpha_{41}\frac{\partial u}{\partial x} + \alpha_{42}u\right)\Big|_{x=l_2+pt} + \left(\alpha_{43}\frac{\partial v}{\partial y} + \alpha_{44}v\right)\Big|_{y=l_4+qt} = h_4(t), \tag{6}$$

where p, q, ℓ_k, α_{jk} (i, k = 1, 2, 3, 4) are assigned positive constants; $\alpha_{11}^2 + \alpha_{21}^2 \neq 0$; $\alpha_{32}^2 + \alpha_{42}^2 \neq 0$; $\alpha_{13}^2 + \alpha_{33}^2 \neq 0$; $\alpha^2{}_{24} + \alpha_{44}^2 \neq 0$; $h_k(t)$ (k = 1, 2, 3, 4) are assigned functions of time satisfying the condition

$$h_k(t)\exp\left(-\frac{t}{2}\right) \in \mathscr{L}_2(-\infty, \infty) \quad (k = 1, 2, 3, 4). \tag{7}$$

As is well known [1, 2], the solution can be represented in the form of sums of the thermal potentials of a single and a double layer:

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